

## Quantum optical coherence in cytoskeletal microtubules: implications for brain function

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### Abstract

'Laser-like,' long-range coherent quantum phenomena may occur biologically within cytoskeletal microtubules. This paper presents a theoretical prediction of the occurrence in biological media of the phenomena which we term 'superradiance' and 'self-induced transparency'. Interactions between the electric dipole field of water molecules confined within the hollow core of microtubules and the quantized electromagnetic radiation field are considered, and microtubules are theorized to play the roles of non-linear coherent optical devices. Superradiance is a specific quantum mechanical ordering phenomenon with characteristic times much shorter than those of thermal interaction. Consequently, optical signalling (and computation) in microtubules would be free from both thermal noise and loss. Superradiant optical computing in networks of microtubules and other cytoskeletal structures may provide a basis for biomolecular cognition and a substrate for consciousness.

**Key words:** Quantum theory; Quantum coherence; Photon coherence; Neural holography; Microtubules; Spontaneous symmetry breaking; Water molecules; Photon signalling network; Consciousness

### 1. Introduction

Several features of brain activity have prompted connections to quantum physics. One feature is that the physical correlates of consciousness,

presumably patterns composed of many states of many entities, are distributed throughout the brain. Despite this extension in space, consciousness has a 'unity' that is difficult to explain by classical means (Marshall, 1989). Non-local quantum coherence may provide the unity necessary to break the impasse known as the 'binding problem' in neurophysiological terms (Crick and Koch, 1990; Singer, 1993).

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Another feature, as argued by Penrose (1989), is that many brain functions are non-algorithmic and non-computational, and thus inexplicable by serial and/or parallel neural processing. Quantum coherence (in microtubules specifically, according to Penrose (1994)) can account for these functions. Other links between consciousness and quantum theory include probabilistic synaptic functions (Eccles, 1986; Beck and Eccles, 1992), holography (Pribram, 1966, 1971, 1991; Gabor, 1968), and general application of the uncertainty principle (Bohm, 1951; Bohr, 1958).

In the mid 1960s the advent of optical holography heralded a new departure in understanding the relationship between brain, memory and perception. The resistance of aspects of memory and of perception to relatively extensive brain damage indicated that memory storage and perceptual processing are distributed procedures. Before the engineering instantiations of Gabor's holographic equations were available, it was difficult to imagine what such a distributed procedure would be like.

Over the next decades computer programs inspired by holography were developed that reflected some of the associative characteristics of the parallel distributed processes that constitute the essence of holographic optical procedures. These 'connectionist', 'neural network' procedures made it possible to stimulate *in vitro* many perceptual and memory processes and to explore the extent and limitations of the simulations.

Meanwhile, it was recognised that an unmodified holographic metaphor was an inappropriate model for brain processing because its spread function is unlimited (infinite). Data from neurophysiological experiments showed that the cortical function modelled by holography is better represented by a sinusoid limited by a Gaussian envelope — a mathematical formulation put forward by Gabor (1946) to measure the maximum efficiency with which a telephone message could be sent across the Atlantic cable. Gabor (1948, 1968) used the same equations used by Heisenberg to describe units in microphysics and, therefore, called the unit of communication a 'quantum of information'.

A major interest confronting brain science is, therefore, to probe the possible mechanisms that lead to the processing of quanta of information by the brain.

A specific quantum field theoretical approach was originated by Ricciardi and Umezawa (1967). Without reference to the structural constituents of brain cells, they formulated a relationship between quantum physics and memory. They emphasized that each neuron specifies a spatially distributed system with quantum mechanical degrees of freedom whose physical properties can be understood by quantum field theory. Quantum field theory is a framework of quantum physics capable of describing fundamental processes of elementary particle physics as well as condensed matter physics (Umezawa, 1993). Ricciardi and Umezawa (1967) suggested that memory is a quantum vacuum state that violates an original dynamical symmetry property of the full complement of the spatially-distributed quantum mechanical degrees of freedom in each brain cell.

The introduction of quantum theory in brain science is perhaps analogous to the recent advent of the new field, quantum cosmology. The fundamental equations of cosmology, recast as quantum theoretical operator expressions acting on a 'wavefunction of the universe' permit an interpretation in which the universe is born in imaginary time. A quantum tunnelling process would then allow the universe to evolve into real time, according to a proposal by Hawking. Similarly, the model advanced by Ricciardi and Umezawa (1967) relies on the quantum theoretical concept of a vacuum state which violates a dynamical symmetry property. While the marriage of quantum theory and cosmology is uneasy so long as quantum theory remains uninterpretable outside the context of a classical environment, the union of brain science with quantum concepts is consistent with currently accepted interpretations. Memory of an external stimulus is imprinted as an ordering of the vacuum state (i.e. the lowest energy state or the ground state). If this ordered vacuum state no longer manifests the original dynamical symmetry property, the Nambu-Goldstone theorem suggests creation of long-range correlation

waves with zero energy requirement which participate in the system's quantum dynamics (Ricciardi and Umezawa, 1967).

Such a long-range correlation wave can be seen as a Bose quantum ('Goldstone boson' or 'massless boson'). Ricciardi and Umezawa were perhaps the first to suggest the existence of a spatially distributed system with the full range of quantum mechanical degrees of freedom in brain cells, and also introduced quantum ordering and spontaneous symmetry breaking as tools to investigate the biological system. Slightly later, Fröhlich (1968) proposed a similar idea for biological cells in general but with more explicit materialistic formulation. Focusing on a thin layer just beneath the cell membrane, Fröhlich theorized that energy can be stored without thermal loss in this two-dimensional region in coherent, dipolar propagating waves. Biological molecules with dipolar vibrational activity adjacent to cell membranes may thus manifest a globally coherent mode of dynamics so that the thin layer could be seen as a biological superconducting medium or a biological plasma effectively isolated from thermal environments. Fröhlich waves are predicted to appear in a frequency region between  $10^{11}$  and  $10^{12}$   $s^{-1}$  called the Fröhlich frequency. It can be concluded that energy supplied to biological cells with magnitude equal to the 'Fröhlich frequency' times the Planck constant may not be completely thermalized but stored in a highly ordered fashion.

Experimental evidence for Fröhlich excitations in biological systems include observation of GHz-range phonons in proteins (Genberg et al., 1991), sharp-resonant non-thermal effects of microwave irradiation on living cells (Grundler and Keilmann, 1983), GHz-induced activation of microtubule pinocytosis in rat brain (Neubauer et al., 1990), and resonance Raman detection of Fröhlich frequency energy (Genzel et al., 1976; Webb et al., 1977; Webb and Stoneham, 1977; Webb, 1980).

Fröhlich waves need not be restricted to thin layers adjacent to cell membranes. Dipolar oscillations maintained by hydrogen bonds and non-localized electrons trapped in hydrophobic regions of protein molecules may manifest a collective

mode considered as a Fröhlich wave propagating in a one-dimensional medium. Davydov described dipolar solitary waves propagating along alpha-helices within proteins. This mode of solitary wave propagation, known in quantum field theory to carry energy and information without loss due to thermalization, is called the Davydov soliton or dipolar soliton (Davydov, 1979).

About 10 years after the original idea of 1967, Umezawa together with two colleagues, Takahashi and Stuart, proposed a refined physical model of a spatially-distributed system with fully quantum mechanical degrees of freedom, not only within each brain cell but also among brain cells (Stuart et al., 1978, 1979). The system extended among brain cells is thought to be composed of two kinds of spatially distributed degrees of freedom subject to quantum dynamics. As a consequence of spontaneous symmetry breaking, these two modes are distinguished as the 'corticon' field and an exchange boson field which interact as a quantum dynamical system — a theoretical model of non-local memory storage and recall processes. More recently, Jibu and Yasue presented a physically realistic picture of the system of corticon and exchange boson fields, and proposed to call the new quantum field theoretical framework 'Quantum Brain Dynamics' ('QBD') (Jibu and Yasue, 1992a,b, 1993a,b). They describe corticon and exchange boson fields as dipolar vibrational fields distributed along protein filaments of the cytoskeleton and extracellular matrices and associated water fields. Marshall (1989) proposed that Fröhlich's 'pumped phonons' result in a Bose-Einstein condensate in brain which is the substrate for consciousness.

Inspired by application of quantum theoretical methods to brain and other biological cells initiated by Ricciardi and Umezawa and by Fröhlich in the 1960s, other scientists began to view brain functioning from the microscopic view of quantum physics during the 1980s. Several groups focused especially on networks of filamentous proteins which dynamically organize the interiors of living cells: the cytoskeleton (Hameroff and Watt, 1982; Del Giudice et al., 1983). Among cytoskeletal filamentous polymers (actin, interme-

diate filaments, etc.), microtubules are the most central to cellular organization and information processing. Consisting of skewed hexagonal lattices of subunit protein dimers called tubulin, microtubules self-assemble into hollow cylinders 25 nm across, adapt, transport and govern cellular activities. Their hollow geometry and periodic lattice structure have suggested several quantum field aspects. Hameroff (1987) applied Fröhlich's coherent excitations to tubulin subunits in microtubules as the basis of computational cellular automata. Rasmussen et al. (1990) showed that networks of such microtubule automata are capable of learning. Considering the layer of ordered water outside and inside microtubules, Del Giudice et al. (1986) proposed that the formation of microtubules' cylindrical structure from 'tubulin' subunits may be understood by the concept of self-focussing of electromagnetic energy by ordered water. Like the Meissner effect for superconducting media, electromagnetic energy would be confined inside filamentous regions around which the tubulin subunits gather. Del Giudice et al. (1986) showed that this self-focussing should result in filamentous beams of radius 15 nm, precisely the inner diameter of microtubules (see Figs 1–4).

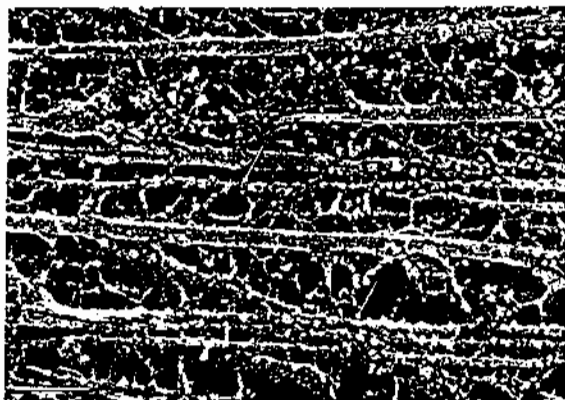


Fig. 1. Electron micrograph of quick frozen, deep, etched neuronal microtubules (MTs) and neurofilaments polymerized with microtubule associated proteins (MAPs). Arrows point at neurofilaments; larger diameter structures are microtubules. Scale bar, 100 nm. (With permission from Hirokawa, 1991).

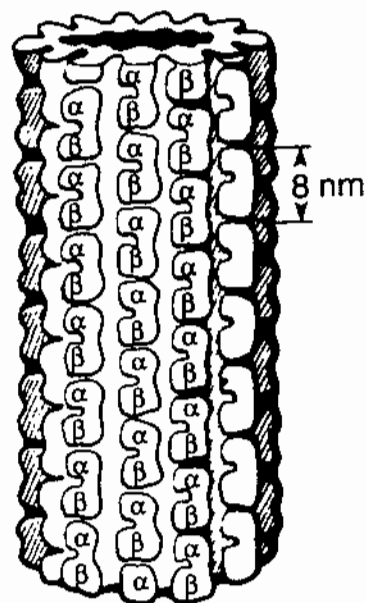


Fig. 2. Microtubule (MT) structure from X-ray crystallography (Amos and Klug, 1974). Tubulin subunits are 8-nm dimers comprised of  $\alpha$  and  $\beta$  monomers.

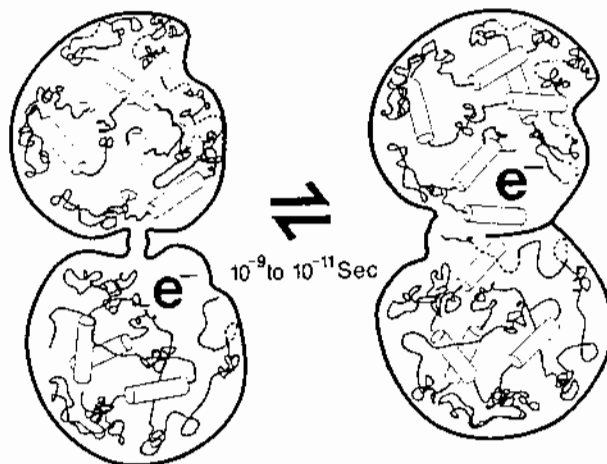


Fig. 3. The tubulin dimer protein has two states in which a quantum event (electron mobility, quasi-particle phonon) within a hydrophobic region couples to conformational change with  $10^{-9}$ – $10^{-11}$ -s transitions. Cylinders within protein are  $\alpha$ -helical regions. (Original drawing by Djuro Koruga).

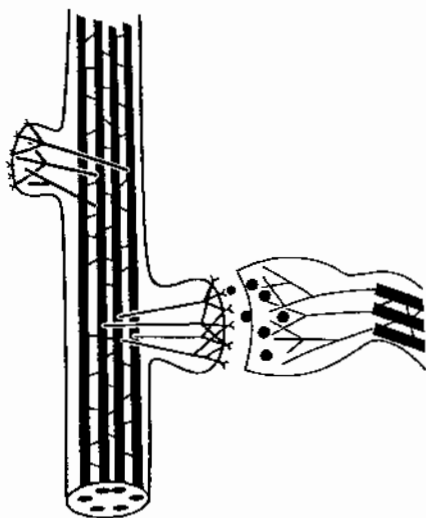


Fig. 4. Schematic drawing of a synapse showing underlying cytoskeletal structures. Right: axon terminal with MT-MAP network connecting with synapsin filaments involved in release of neurotransmitter vesicles (circles) into synaptic cleft. Left: ventricular dendrite with MT-MAP network, two dendritic spines with cytoskeletal structures connected to receptors in synaptic membrane. (Adapted from Hirokawa, 1991).

Another perspective was proposed by Hameroff (1974) in which microtubules were thought to act like 'dielectric waveguides' for photons, that is, quantum dynamical modes of an electromagnetic wave. Indeed, living tissue transmits light more readily than non-living material; in experiments with mammalian brains, temporal poles and hippocampus manifest the maximum light penetration (Hameroff, 1987). This evidence led Hameroff to consider microtubules as waveguides for photons. Furthermore, he proposed that cytoplasmic interference of coherent sources from and among multiple microtubules may lead to a holographic information processing mechanisms. Holographic patterns stored in cytoplasmic sol-gel states may become 'hardwired' by calcium-induced assembly of finer components of the cytoskeleton such as actin, actin-binding proteins and the micro-trabecular lattice. Lechleiter et al. (1991) imaged coherent cytoplasmic spiral waves of calcium induced by acetylcholine binding to membrane receptors of living cells. Coherent

sources of either charge carriers or conformational waves in microtubules suggest a revival of holographic brain theory of Pribram and others (Pribram, 1966, 1971, 1991; Gabor, 1968; Margenau, 1984; Eccles, 1986; Marshal, 1989). Our current view is that coherence manifests at the molecular, quantum level rather than (or in addition to) coherence at the level of neural firing (Singer, 1993).

Hameroff suggested in very general terms two key concepts in understanding cytoskeletal brain activity from the point of view of quantum physics: microtubules acting as waveguides for photons and as holographic information processors. Holography is an optical phenomenon related to interference of coherent electromagnetic waves or photons. The microtubule may be seen as a waveguide for photons, and Fröhlich's theory suggests coherent excitations in microtubules. Further, the periodic lattice structure of microtubules may provide periodically arrayed 'slits' (spaces between dimers) through which photons may pass. No proof of coherent photon generation or emission necessary for optical holography has been found in microtubules or any microscopic biological structure. However, self-focused photons and self-trapped, non-thermalizing wave entities should be difficult to detect (see Section 4). In any case, no thorough theoretical investigation of these concepts has occurred from the standpoint of physics except the water laser (Del Giudice et al., 1988).

In the present paper, we propose a new quantum theoretical framework suggesting microtubules possess ordered and systematic properties yielding cooperative quantum dynamics called superradiance and self-induced transparency. Specifically, the quantum dynamical system of water molecules and the quantized electromagnetic field confined inside the hollow microtubule core can manifest a specific collective dynamic called superradiance by which the microtubule can transform any incoherent, thermal and disordered molecular, electromagnetic or atomic energy into coherent photons inside the microtubule. Furthermore, it will also be shown that such coherent photons created by superradiance penetrate perfectly along the internal hollow core of the microtubule as if

the optical medium inside it were made 'transparent' by the propagating photons themselves. This is a quantum theoretical phenomenon called self-induced transparency.

Superradiance and self-induced transparency in cytoskeletal microtubules can lead to 'optical' neural holography. Neurons (and other cells) may contain microscopic coherent optical supercomputers with enormous capacity like the fictional but famous HAL 9000 of '2001: A Space Odyssey' with holographic memory manipulation (Clarke, 1968).

This paper is a result of the long-range cooperation among physicists and brain scientists providing several by-products:

1. Superradiance and self-induced transparency of microtubules may be understood to predict theoretically the existence of a peculiar phenomenon: weak coherent photon emission from living matter.
2. Quantum coherence among microtubules spaced hundreds of micrometers apart (which in turn can have quantum coherence with other microtubules hundreds of micrometers away, etc.) can solve the unity and binding problems which plague classical theories of consciousness.
3. A source of coherence at the biomolecular level can lead to quantum interference and holographic information representation at the sub-cellular level.
4. General anesthesia may be explained by blockade of quantum level events which affect collective cooperative macro-level events.

## 2. Superradiance in microtubules

Let us look at a typical microtubule in the cytoskeletal structure of brain cells or general biological cells. It is a hollow cylinder about 25 nm in diameter whose wall is a polymerized array of protein subunits ('tubulins'). Its length may range from tens of nanometers to micrometers, and possibly further to meters in nerve axons of large animals. For the purpose of simplifying the physical formulation, we consider the microtubule as a hollow cylinder with radius  $r_{MT}$  and length  $l_{MT}$ .

Their real values would be  $r_{MT} \approx 12$  and  $l_{MT} \approx 10^2$ – $10^3$  in nanometers. We denote the spatial region inside the microtubule cylinder by  $V$  and restrict our discussion to quantum mechanical dynamics taking place in this spatial region  $V$ .

Let us introduce a Cartesian system of coordinates  $O_{xyz}$  with the  $xy$ -plane attached parallel to one of the two ends of the microtubule cylinder so that the origin  $O$  coincides with the centre of the end cap. The  $z$ -axis lies naturally along the longitudinal center axis of the microtubule cylinder. Then, any position in the region  $V$  can be labelled by giving its coordinates  $r = (x, y, z)$ .

The spatial region  $V$  inside the microtubule is not empty, but is likely to be filled with water molecules. Of course, there may be other molecules, although their population is relatively small. We consider the ideal case in which the existence of molecules other than those of water can be neglected. However, below we will consider the more realistic case in which we have impurities among water molecules such as anesthetic molecules. It is most likely that the density of water confined inside the microtubule cylinder remains almost constant. Therefore, we may be allowed to fix the total number of water molecules inside the region  $V$ , say  $N$ .

Let us take a look at a typical water molecule, say the  $j$ th water molecule. Here,  $j$  running from 1 to  $N$  denotes the fictitious number labelling the  $N$  water molecules in question. Its position is given by coordinates  $r^j = (x^j, y^j, z^j)$ . From a physical point of view, a water molecule has a constant electric dipole, and so it can be seen as a quantum mechanical spinning top with an electric dipole moment. The average moment of inertia and electric dipole moment of a water molecule are estimated to be  $I = 2m_p d^2$  with  $d \approx 0.82 \text{ \AA}$  and  $\mu = 2e_p P$  with  $P \approx 0.2 \text{ \AA}$ , respectively. Here,  $m_p$  denotes the proton mass and  $e_p$  the proton charge.

Due to the electric dipole moment  $\mu$ , the water molecule interacts strongly with the quantized electromagnetic field in the spatial region  $V$ . Although the water molecule has many energy eigenstates as a quantum mechanical spinning top and so it can exchange energy between the quantized electromagnetic field in many different

values, we restrict our discussion to the most likely case in which only the two principal energy eigenstates take part in the energy exchange. This coincides with the conventional two-level approximation in describing the energy exchange between atoms and the quantized electromagnetic field in laser theory.

Then, one sees immediately that the quantum dynamics of the  $j$ th water molecule can be well described by a spin variable  $s^j = 1/2\sigma$ , where  $\sigma = (\sigma_x, \sigma_y, \sigma_z)$  and the  $\sigma_x$  are Pauli spin matrices denoting the three components of the angular momentum for spin  $1/2$ . Let  $\epsilon$  be the energy difference between the two principal energy eigenstates of the water molecule. Its real value is  $\epsilon \approx 200 \text{ cm}^{-1}$  (Franks, 1972). Then, the Hamiltonian governing the quantum dynamics of the  $j$ th water molecule is given by  $\epsilon s_z^j$ , and so the total Hamiltonian for  $N$  water molecules becomes

$$H_{WM} = \epsilon \sum_{j=1}^N s_z^j \quad (1)$$

Two energy eigenvalues of the former Hamiltonian are  $-1/2\epsilon$  and  $1/2\epsilon$  reflecting the fact that only the two principal energy eigenstates with energy difference  $\epsilon$  are taken into account.

Now, let us consider the quantized electromagnetic field in the spatial region  $V$ . It is convenient to describe the quantized electromagnetic field in terms of an electric field operator  $E = E(r, t)$ . Let us assume for simplicity that the electric field is linearly polarized, obtaining  $E = eE$ , where  $e$  is a constant vector of unit length pointing in the direction of linear polarization. Then, the quantized electromagnetic field in question comes to be well described by a scalar electric field  $E = E(r, t)$  governed by the usual Hamiltonian

$$H_{EM} = \frac{1}{2} \int_V E^2 d^3r \quad (2)$$

Next, we take the interaction between the quantized electromagnetic field and the totality of water molecules by which they can exchange energy in terms of creation and annihilation of photons. Let

us divide the electric field operator into positive and negative frequency parts

$$E = E^+ + E^- \quad (3)$$

Then, the interaction Hamiltonian of the quantized electromagnetic field and the totality of water molecules becomes

$$H_I = -\mu \sum_{j=1}^N \{ E^-(r^j, t) s_-^j + s_+^j E^+(r^j, t) \} \quad (4)$$

where

$$s_{\pm}^j = s_x^j \pm i s_y^j \quad (5)$$

The total Hamiltonian governing the quantum dynamics of the electromagnetic field, the dipolar vibrational field of water molecules, and their interaction is given by

$$H = H_{EM} + H_{WM} + H_I \quad (6)$$

Since the spatial region  $V$  inside the microtubule cylinder may be considered as a cavity for the electromagnetic wave, it is convenient to introduce the normal mode expansion of the electric field operator  $E = E^+ + E^-$ , obtaining

$$E^{\pm}(r, t) = \sum_k E_k^{\pm}(t) e^{\pm i(k \cdot r - \omega_k t)} \quad (7)$$

Here,  $\omega_k$  denotes the proper angular frequency of the normal mode with wave vector  $k$ . We are mainly interested in the ordered collective behavior among the water molecules and the quantized electromagnetic field in the cavity region  $V$ . Let us introduce hence collective dynamical variables  $S_k^{\pm}(t)$  and  $S$  for water molecules by

$$S_k^{\pm}(t) = \sum_{j=1}^N s_{\pm}^j(t) e^{\pm i(k \cdot r^j - \omega_k t)} \quad (8)$$

and

$$S = \sum_{j=1}^N s_z^j \quad (9)$$

Then, the total Hamiltonian (Eq. 6) becomes

$$H = H_{EM} + \epsilon S - \mu \sum_k (E_k^- S_k^- + S_k^+ E_k^+) \quad (10)$$

It seems worthwhile to note here that this total Hamiltonian for the system of  $N$  water molecules and the quantized electromagnetic field in the region  $V$  inside the microtubule cylinder is essentially of the same form as not only Dicke's Hamiltonian for the laser system but also that of Stuart et al. for Quantum Brain Dynamics (Dicke, 1954; Stuart et al., 1979). Therefore, it can be expected that each microtubule in the cytoskeletal structure of brain cells manifests not only the memory printing and recalling mechanism in QBD but also a laser-like coherent optical activity. The former seems not so surprising because Jibu and Yasue (1992a, 1993a,b) developed a physical picture of QBD in terms of water molecules and protein filaments. Hence, we will not discuss this in the present paper. On the other hand, the latter seems highly surprising because it may open a completely new picture of the fundamental process of brain functioning, drastically different from the conventional one. Namely, in addition to the usual pathway conduction of neural impulses in terms of transmembrane ionic diffusions among nerve cells, the brain system may possibly use another but much more microscopic and elaborated fundamental physical process in terms of coherent photon emission and transfer in the microtubule just like the optical computer with lasers, optical fibers and other optical devices. It is this very surprising possibility we are going to discuss throughout the present paper.

However, it is difficult to look for the possibility of realizing laser devices in brain cells even though the microtubule may manifest the same quantum dynamical behavior, in terms of water molecules and the electromagnetic field, as is governed by the Hamiltonian of a typical laser system. This is because we need initially to pump up the majority of water molecules to the higher energy eigenstate by certain incoherent but high intensity light. In actual artificial laser devices, for example, the initial pumping is due to xenon flash lamps. In other words, the laser system cannot emit coherent

photons without some pumping mechanism. We can hardly expect the existence of such a pumping light in the brain. We are, therefore, forced to look for another possibility with the total Hamiltonian (Eq. 10) leading to a different mechanism of coherent photon emission without pumping light.

Fortunately, a physical inspection of the form of the total Hamiltonian (Eq. 10) reveals that it manifests a dynamical symmetry property not evident in the ground state and so the resulting quantum dynamics is known to involve certain long-range order creating phenomena due to spontaneous symmetry breaking (Stuart et al., 1979). The spatial dimension of this long-range order, that is, the coherence length  $l_c$  can be estimated to be inversely proportional to the energy difference  $\epsilon$ , obtaining  $l_c \approx$  hundreds of micrometers. Among those long-range order creating phenomena we may find a specific one in which the collective dynamics of the majority of water molecules inside the microtubule cylinder  $V$  can give rise to cooperative spontaneous emission of photons without any pumping light. Any incoherent and disordered energy distribution among the water molecules due to the macroscopic thermal dynamics of the polymerized array of protein subunits (i.e. tubulins) forming the wall of microtubule cylinder can be gathered collectively into coherent and ordered dynamics ready to emit coherent photons cooperatively. This laser-like process of coherent photon emission without pumping light was first introduced by Dicke (1954) and called superradiance.

Let us investigate the superradiance in the microtubule cylinder  $V$  starting from the total Hamiltonian (Eq. 10). We assume for simplicity that only one normal mode with a specific wave vector, say  $k_0$ , has a proper angular frequency  $\omega_{k_0}$ , resonating to the energy difference  $\epsilon$  between the two principal energy eigenstates. Namely we have

$$\epsilon = \hbar\omega_{k_0} \quad (11)$$

and all the other normal modes are neglected. In the conventional laser theory, this is known as a single mode laser. What we analyse below may hence be considered as a single mode superradiance in the microtubule.

Since we have only one normal mode with wave



vector  $k_0$ , we may omit all the wave vector indices of the dynamical variables. Then, the total Hamiltonian (Eq. 10) becomes

$$H = H_{EM} + \epsilon S - \mu(E^- S^- + S^+ E^+). \quad (12)$$

The corresponding Heisenberg equations of motion for the three collective dynamical variables,  $S$  and  $S^\pm$ , for water molecules and the two variables,  $E^\pm$ , for the quantized electromagnetic field are given by:

$$\frac{dS}{dt} = -i \frac{\mu}{\hbar} (E^- S^- - S^+ E^+) \quad (13)$$

$$\frac{dS^\pm}{dt} = \pm i \frac{2\mu}{\hbar} S E^\mp \pm i \frac{\epsilon}{\hbar} S^\pm \quad (14)$$

and

$$\frac{dE^\pm}{dt} = \pm i \frac{2\pi\epsilon\mu}{\hbar V} S^\mp \quad (15)$$

Because of the short length of the microtubule cylinder —  $l_{MT} \approx 10^2-10^3$  nm — the pulse mode propagating along the microtubule cylinder in the direction of the  $z$ -axis stays in the cavity region  $V$  only for a short transit time  $t_{MT} \equiv l_{MT}/c$ , where  $c$  stands for the speed of light. As this transit time of the pulse mode is much shorter than the characteristic time of thermal interaction due to the disordered environment, the system of water molecules and quantized electromagnetic field in this single mode superradiance is free from thermal loss and can be considered as a closed system well-described by the Heisenberg equations of motion (Eqs. 13-15). Furthermore, the time derivative of the dynamical variables  $E^\pm$  of the quantized electromagnetic field can be approximated by  $E^\pm/t_{MT}$  in the case of a pulse mode propagating along the longitudinal axis of the microtubule. Eq. 15 then yields

$$E^\pm = \pm i \frac{2\pi\epsilon\mu l_{MT}}{\hbar V} S^\mp \quad (16)$$

This means that a pulse mode of the quantized electromagnetic field in the microtubule cylinder cavity follows the collective dynamics of water molecules inside the microtubule. In other words, once a collective mode with long-range order is created in the dynamics of water molecules due to spontaneous symmetry breaking, coherent emission of pulse modes of the quantized electromagnetic field — that is, photons — follows. This is the mechanism of superradiance.

The last question is whether such a collective mode can be realized in the dynamics of water molecules starting from incoherent and disordered initial conditions. Notice that such an incoherent and disordered initial dynamical configuration of water molecules is due to the interaction between water molecules and thermally disordered states of the conformational dynamics of tubulins. The onset of this collective mode can be seen by rewriting the three Heisenberg equations for the collective variables of water molecules (Eqs. 13 and 14) by substituting Eq. 16. Namely, we have

$$\frac{dS^\pm}{dt} = \beta S S^\pm \pm i \epsilon S^\pm \quad (17)$$

and

$$\frac{dS}{dt} = -\beta S^+ S^- \quad (18)$$

where

$$\beta = \frac{4\pi\epsilon\mu^2 l_{MT}}{\hbar^2 V}.$$

These are coupled non-linear differential equations for non-commuting operators  $S$  and  $S^\pm$  subject to certain commutation relations, and it is not so easy to find their solutions. However, if we regard these equations as ordinary coupled non-linear differential equations for classical (i.e. commuting) dynamical variables, we can find special solutions with respect to any incoherent and disordered initial conditions. They are known as semi-classical approximations of the quantum

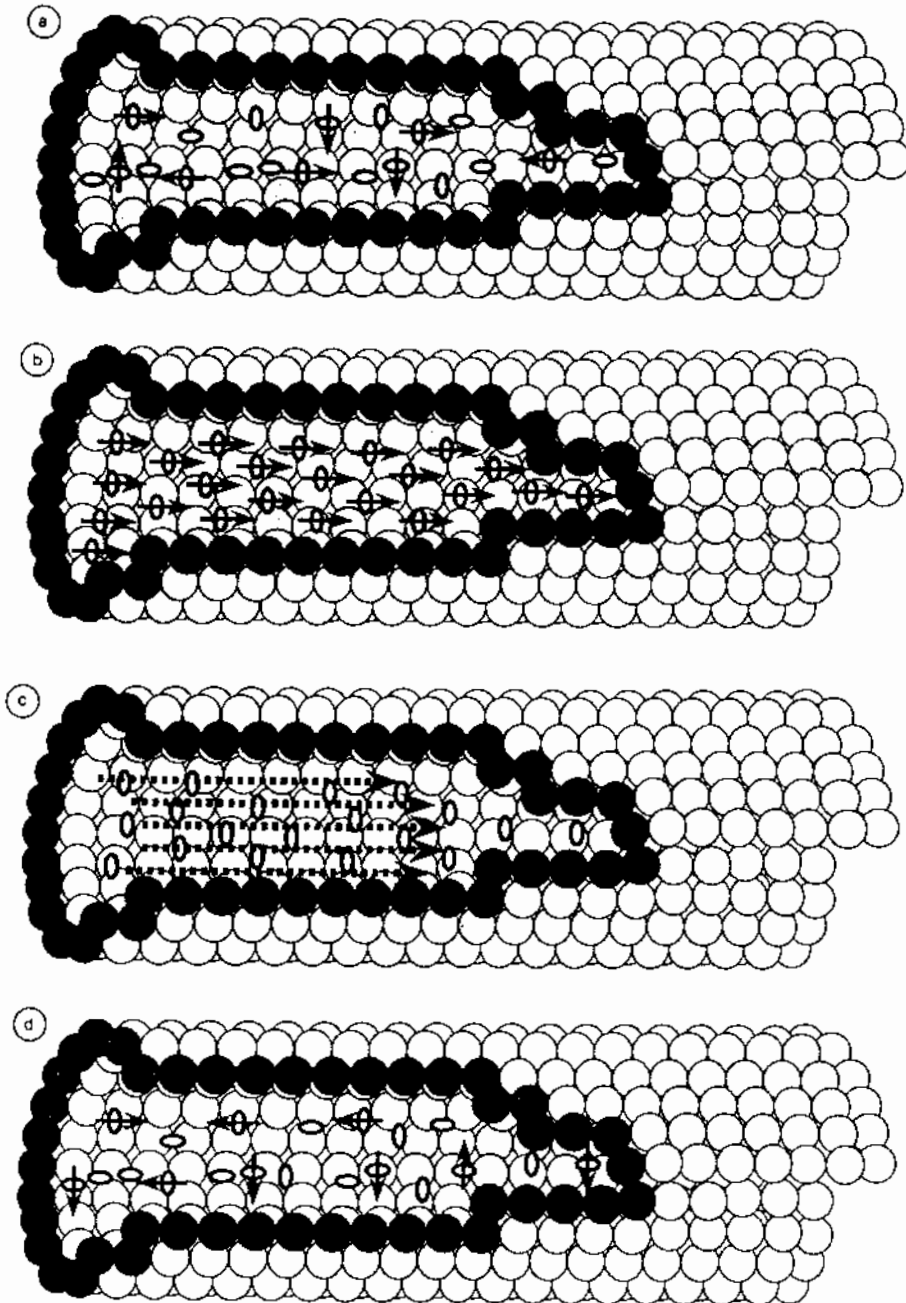


Fig. 5. A schematic representation of the process of superradiance in a microtubule. Each oval without an arrow stands for water molecule in the lowest rotational energy state. Each oval with an arrow stands for a water molecule in the first excited rotational energy state. The process is cyclic (a → b → c → d → a → b), and so on. (a) Initial state of the system of water molecules in a microtubule. Energy gain due to the thermal fluctuation of tubulins increases the number of water molecules in the first excited rotational energy state. (b) A collective mode of the system of water molecules in rotationally excited states. A long-range coherence is achieved inside a microtubule by means of spontaneous symmetry breaking. (c) A collective mode of the system of water molecules in rotationally excited states loses its energy collectively, and creates coherent photons in the quantized electromagnetic field inside a microtubule. (d) Water molecules, having lost their first excited rotational energies by superradiance, start again to gain energy from the thermal fluctuation of tubulins, and the system of water molecules recover the initial state (a).

dynamical system of superradiance (Agarwal, 1971). A straightforward calculation yields that the intensity of coherent photon emission in the microtubule cylinder due to superradiance can be given in this approximation by

$$I = \frac{\hbar^2}{(4t_R\mu)^2} \operatorname{sech}^2 \frac{t-t_0}{2t_R} \quad (19)$$

where

$$t_R = \frac{c \hbar^2 V}{4\pi\mu^2 \epsilon N l_{MT}}$$

and  $t_0 = t_R \ln 2N$  denotes the life time and delay of the superradiance, respectively. Notice that the intensity of superradiance is proportional to  $N^2$  and its delay time is inversely proportional to  $N$ . These facts are characteristic to the long-range order creating process involving  $N$  water molecules (see Fig. 5).

We have found that the quantum collective dynamics of water molecules and a quantized electromagnetic field inside the microtubule cylinder manifests the long-range cooperative phenomenon of superradiance in which collective excitation of water molecules can be induced by incoherent and disordered perturbations due to the macroscopic thermal dynamics of protein molecules forming the wall of the microtubule cylinder. This fact ensures that each microtubule in the cytoskeletal structure of brain cells, that is, neurons and astrocytes, may play an important role in the optical information processing regimen of brain function as a superradiant device which converts the macroscopic disordered dynamics of water molecules and protein molecules into the long-range ordered dynamics of water molecules and a quantized electromagnetic field involving a pulse mode emission of coherent photons. In other words, each microtubule is a coherent optical encoder in a dense microscopic optical computing network in the cytoplasm of each brain cell, if such a network is realized in actual cytoplasmic structure. This last point is far from evident and deserves to be discussed in the following section.

### 3. Self-induced transparency in microtubules

We have shown the possibility of a completely new mechanism of fundamental brain functioning in terms of coherent photon emission by superradiance in microtubules. Unlike a laser, superradiance is a specific quantum mechanical ordering process with a characteristic time much shorter than that of thermal interaction. Therefore, microtubules may be thought of as ideal optical encoders providing a physical interface between (1) the conventional macroscopic system of classical, disordered and incoherent neural dynamics in terms of transmembrane ionic diffusions as well as thermally perturbed molecular vibrations and (2) the as yet unknown microscopic optical computing network system of ordered and coherent quantum dynamics free from thermal noise and loss. As Feynman (1985) proposed, the optical computing network is the most realizable quantum mechanical computer among many possibilities such as a superconducting computer. However, it is not clear whether the pulse mode coherent photons created in the microtubule cylinder by superradiance can be safely transmitted, preserving its long-range coherence. It seems most likely that even coherent photons emitted by superradiance will lose immediately their coherence and long-range order due to the noisy thermal environment. In any case, we absolutely require some quantum dynamical mechanism to maintain the coherent transmission of photons in the as yet unknown microscopic optical computing network in brain cells. In this section, we will show that it is again the microtubule in the cytoskeletal structure of brain cells which provides us with such a mechanism.

Let us suppose that the pulse mode coherent photons are created in a small segment of the microtubule cylinder by superradiance. Then, those photons propagate along the longitudinal axis of the microtubule cylinder, that is, the  $z$ -axis. If the region  $V$  inside the microtubule cylinder were maintained at vacuum, they would transmit through the region just as they do along a waveguide. However, the region is filled up with water molecules, and it is not evident at all whether the pulse mode coherent photons can be

safely transmitted through the region without absorption or loss of coherence.

For the purpose of describing the transmission of pulse mode coherent photons along the region inside the microtubule cylinder, it seems convenient to work with a semi-classical approximation in which the long-range ordered dynamics of water molecules is described by quantum mechanics but the electromagnetic field is described classically. This approximation becomes reliable when the intensity of the pulse mode coherent photons is large, and this is indeed the case for superradiance emission (Feynman et al., 1957).

Let us consider the Maxwell equation for the scalar electric field  $E = E(z, t)$ , standing for the pulse mode coherent photons propagating along the  $z$ -axis coupled to the collective dynamical variables  $S^\pm$  of water molecules inside the microtubule cylinder,

$$\frac{\partial E^\pm}{\partial z} + \frac{\partial E^\pm}{\partial t} = \mp i \frac{2\pi\epsilon\mu}{\hbar V} S^\mp \quad (20)$$

This equation is valid under the condition that the collective dynamical variables  $S^\pm$  are slowly varying, that is,

$$\frac{\partial S^\pm}{\partial t} \ll i\omega S^\pm \quad (21)$$

Then, taking the expectation of those quantum mechanical variables in this Maxwell equation and the Heisenberg equations of motion (Eqs. 13 and 14), eliminating all the expectation values of those variables referring to water molecules, and introducing new variables for the scalar electric field by

$$\theta^\pm(z, t) = \frac{2\mu}{\hbar} \int_{-\infty}^t E^\pm(z, u) du \quad (22)$$

we can obtain dynamical equations for the electromagnetic field in the region  $V$  inside the microtubule cylinder

$$\frac{\partial^2 \theta^\pm}{\partial \tau \partial \zeta} = -\sin \theta^\pm \quad (23)$$

Here,

$$\tau = \sqrt{\frac{2\pi\epsilon\mu^2 N}{\hbar^2 V}} \left( t - \frac{z}{c} \right)$$

and

$$\zeta = \sqrt{\frac{2\pi\epsilon\mu^2 N}{\hbar^2 V}} \frac{z}{c}$$

This is a typical non-linear partial differential equation called the Sine-Gordon equation, and several exact solutions are obtained by means of the inverse scattering method of soliton equations (Ablovitz et al., 1974).

The most interesting solution of the Sine-Gordon equation (Eq. 23) gives rise to an explicit form of the time evolution of the scalar electric field  $E$  in the region  $V$  inside the microtubule cylinder

$$E = \sqrt{\frac{2\pi\epsilon N v_0}{V(c-v_0)}} \operatorname{sech} \sqrt{\frac{2\pi\epsilon\mu^2 N v_0}{\hbar^2 V(c-v_0)}} \left( t - \frac{z}{v_0} \right) \quad (24)$$

This is nothing but a soliton solution and tells us that the pulse mode photons propagate along the dielectric waveguide of the microtubule cylinder filled up with water molecules with a certain constant speed  $v_0$  less than the speed of light in vacuum  $c$ . It is important to see that the pulse form of the soliton solution is kept unchanged due to the non-linearity of the Sine-Gordon equation.

We have found that microtubules play the role of dielectric waveguides and that pulse mode coherent photons propagate through them as if they were perfectly transparent. This phenomenon is termed self-induced transparency and known to be a typical non-linear effect in quantum optics (McCall and Hahn, 1967). The microtubule may be an ideal microscopic optical device for use as a perfectly transparent pathway for pulse mode photons, free from thermal noise and loss. Combined with superradiance, this self-induced transparency of the microtubule allows us to conclude that the brain is essentially a dense assembly of

microscopic and elaborated optical computing networks of microtubules in the cytoskeletal structure of brain cells. Coherent photon emission and transfer in each microtubule are ensured by superradiance and self-induced transparency characteristic to long-range ordering phenomena in quantum dynamics.

Both superradiance and self-induced transparency may be essential to cooperative physical activity of multiple microtubules in cytoskeletal structure in single cells and among many cells. Thus biological systems in general and the brain in particular may utilise a quantum level holographic information system (Hameroff, 1987; Pribram, 1991).

#### 4. Implications of brain function

We have developed a theoretical framework for the occurrence of quantum coherence of photons in microtubules and their associated water. Fundamental involvement of microtubules and the cytoskeleton in orchestration and control of 'real time' cell functions suggests that such quantum coherence, if it exists, could play a major role in cellular communication, signalling and information processing (Conrad, 1988).

Several aspects of quantum coherence in microtubules may be considered:

1. Superradiance and self-induced transparency occurring in ordered water within the hollow core of cylindrical microtubules behaving as waveguides will result in coherent photons. This coherence, estimated to be capable of superposition of states among microtubules spatially distributed over hundreds of micrometers, which in turn are in superposition with other microtubules hundreds of micrometers away in other directions and so on, could account for a coupling of microtubule dynamics over wide areas. This in turn could account for a unity of thought and consciousness. For example, Insinna (1992) has attributed to quantum coherence in microtubules the phenomenon of Jungian synchronicity.
2. Coherent interference leading to holographic

information representation would result from quantum optical effects. At the cytoskeletal level, this could occur via calcium induced sol-gel variations causing polymerization of actin or other cytoskeletal filaments resulting in a holographic representation of short-term memory (Hameroff, 1987). The holographic paradigm suggests coherence and holographic interference across multiple scales (Pribram, 1991). At a more 'macro' level, coherence of neuronal firing among widely dispersed neurons in the 40-Hz range may be related to quantum coherence of microtubules within coherently firing neurons (Singer, 1993).

3. Sequelae of quantum events in individual microtubule subunits, acoustoconformational vibrations could propagate as signals through cytoskeletal networks and membranes and extracellular matrices. Such proposed signals (in the context of the cellular automata theory) have been shown theoretically capable of learning, logic and adaptation (Rasmussen et al., 1990; Lahoz-Beltra et al., 1993).

We have described a complex energy/information field originating in cytoskeleton and associated water and cooperatively pervading other biostructures. 'Unifying field theories' have a long and unsuccessful history in biology and biophysics and connote a derisive term: 'vitalism'. We introduce a neo-vitalistic, quantum coherent holographic field based on microtubule-derived Goldstone bosons as a medium of biological information, communication and representation of external world. In its largest 'brain-wide' scale, this should constitute consciousness.

Fueled by biochemical energy from ATP and GTP hydrolysis and protein phosphorylation, microtubule quantum coherence and self-focussing should be sensitive to general anesthesia. Anesthetic gas molecules reversibly inhibit consciousness by weak, van der Waals binding in hydrophobic regions of proteins (Franks and Lieb, 1982). If quantum optical coherence in microtubules is essential for consciousness, anesthetics must somehow inhibit it. One possible way is by membrane inhibition which could disconnect and isolate cytoskeletal dynamics from the external

milieu. Alternatively, anesthetics may directly alter cytoskeletal quantum coherence. General anesthetics do bind to microtubules and at high enough concentrations cause depolymerization (Allison and Nunn, 1968). At concentrations of anesthesia just sufficient for ablation of consciousness, inhibition of tubulin conformational dynamics could have profound effects. Wulf and Featherstone (1957) showed that anesthetic binding within protein hydrophobic regions altered protein-water binding at the protein surface. Thus, slight anesthetic effects on tubulin conformational dynamics and cooperative water binding would have significant effects on quantum coherence. Anesthesia is also known to be reversed by pressure (Halsey, 1976). By condensing a non-linear optical medium, pressure increases quantum cooperative coherent phenomena. Thus our model is consistent with the mechanism of general anesthesia.

## 5. Summary

This paper presents a theoretical framework for quantum optical coherence in cytoskeletal microtubules. Specifically, the quantum dynamical system of water molecules and the quantized electromagnetic field confined inside the hollow microtubule core manifests a specific collective dynamics called superradiance by which coherent photons are created inside the microtubule. Furthermore, such coherent photons created by superradiance penetrate perfectly along the internal hollow core of the microtubule as if the optical medium inside it were made transparent by the propagating photons themselves. This quantum effect is called self-induced transparency. Such effects can account for cellular signalling, holographic information processing and unity of consciousness.

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